Studies of roll waves and slug flow were performed primarily in connection with the mechanism of instability of uniform flow on a steep slope. Uniform flow will become unstable when the velocity of flow is very high or the channel slope is very steep. When this happens, the free surface will form a series of roll waves. In 1945, Vedernikov developed a criterion, Vedernikov number, to identify if the uniform flow is stable. Vedernikov number (N_v) is defined as:

$$Nv = x(1 - R\frac{dP}{dA})F \tag{749}$$

Where: x = 2/3 for Manning's equation

R = hydraulic radius

dP= change in wetted perimeter

dA= change in flow area

F = Froude Number

Where:

$$R\frac{dp}{dA} = \frac{by}{2y + b}\frac{d(2y + b)}{d(by)} = \frac{2Y^*}{2Y^* + 1} \tag{750}$$

Where b = channel bottom width

y = flow depth

Y* = y/b

To be a stable uniform flow, Nv shall be less than or equal to unity.

$$Nv = \frac{2}{3}(1 - \frac{2Y^*}{2Y^* + 1})F \tag{751}$$

$$and\ Nv \le 1.0$$

The limiting Froude number for having a stable uniform flow in a rectangular channel is:

$$F \leq \frac{3}{2}\left(2Y^{*} + 1\right) \qquad (752)$$

This conclusion agrees with the straight line on Plate B-7 of EM 111 O-2-1 601.

Similarly, limiting Froude numbers for trapezoidal channel with various side slopes, z, can be derived as:

$$F \leq \frac{3}{2}\left[\frac{(1 \text{ t } 2kY^{*})(1 \text{ t } 2zY^{*})}{1 + 2zY^{*} \text{ t } 2kzY^{*2}}\right] \text{ in which } k = \sqrt{1+z^{2}} \qquad (753)$$

Care has to be taken when designing a steep channel. Selections of y/b ratio, channel lining roughness and slope shall satisfy the above criteria to avoid roll waves. Otherwise mitigation shall be provided, including additional freeboard or rougher linings.

The height of roll waves can be approximated using the model of positive surges which have an advancing front with the profile similar to a moving hydraulic jump. When the height of the surge is small, the surge appears undular like an undular jump. When the height is increasing, the undulation will eventually disappear and the surge will have a sharp and steep front. Such an unsteady flow can be converted to a steady pattern by adding the wave speed to the flow field. Let the subscript 2 represents the design flow condition determined by Manning's formula for the selected channel cross section and the subscript 1 represents the section without roll waves defined by the limiting Froude number. Solving the continuity and momentum principles simultaneously yields:

$$V_2 = \frac{(V_1 - V_w)A_1 + V_w A_2}{A_2} \qquad (754)$$

and the wave speed in a moving jump is:

$$V_w = V_1 + \sqrt{\frac{(A_2\bar{y}_2 - A_1\bar{y}_1)g}{A_1(1 - A_1/A_2)}} \qquad (755)$$

$$h = y_2 - y_1 \qquad (756)$$

in which V_w = wave velocity, V = flow velocity, A = flow area, g = gravitational acceleration, h = height of roll waves, and $\bar{y}$ = distance to centroid of flow area, approximated by 0.5y. Considering that the roll waves near the center of the channel section is similar to that in a rectangular channel, the height of roll waves can be derived as

$$h = \frac{C^2}{g}\left(\frac{2y_1}{y_1 + y_2}\right)\left(\frac{V_2}{C} - \frac{V_1}{C}\right) = \frac{C^2}{g}\left(\frac{2y_1}{y_1 + y_2}\right)(F_2 - F_1) \qquad (757)$$

$$C = V_w - V_2 \qquad (758)$$

in which F_2 = Froude number for design discharge, F, = limiting Froude number determined by Vedernikov's number, and C = celerity of wave. When the height of roll waves is small compared with the depth of flow, i.e. $y_1 \approx y_2$, Eq 757 is reduced to:

$$h = \frac{C^2}{g}(F_2 - F_1) \qquad (759)$$

The above procedure predicts the height of roll waves when the design condition, F_2, deviates from the limiting condition, F,. It is suggested that design of channel freeboard must include the considerations of roll waves. Engineers must assure that the current design criteria for freeboard provides adequate height to accommodate roll waves on top of super-elevation. Otherwise, additional freeboard shall be added to the channel depth.

Design Freeboard = Maximum (Roll Waves or Empirical Freeboard)

A design example was developed to examine how this criteria will impact the selection of the channel cross-section. For instance, channel bottom widths of 10, 15, and 20 feet are considered to design a channel to pass 5,000 cubic feet per second on a slope of 3.0 percent with Manning's "n" of 0.014. The following table summarizes the recommended channel depth under the consideration of roll waves.